AN EFFICIENT RELAXATION BASED *DIPIE* ALGORITHM FOR COMPUTER AIDED DESIGN OF ELECTROSTATIC ACTUATORS

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ABSTRACT

Pull-In parameters are important properties of electrostatic actuators. Efficient and accurate analysis tool that can capture these parameters for different design geometries, are therefore essential. Current simulation tools approach the Pull-In state by iteratively adjusting the voltage applied across the actuator electrodes. The convergence rate of this scheme gradually deteriorates as the Pull-In state is approached. Moreover, the convergence is inconsistent and requires many mesh and accuracy refinements to assure reliable predictions. As a result, the design procedure of electrostatically actuated MEMS devices can be time-consuming. In this paper a novel Displacement Iteration Pull-In Extraction (*DIPIE*) scheme is presented. The *DIPIE* scheme is shown to converge consistently and far more rapidly than the voltage iterations (*VI*) scheme (>100 times faster!). A relaxation based *DIPIE* scheme that requires separate mechanical and electrostatic field solvers is suggested. Therefore, it can be easily implemented into existing MOEMS CAD packages. Moreover, using the *DIPIE* scheme, the Pull-In parameters extraction can be performed in a fully automated mode, and no user input for search bounds is required.

INTRODUCTION

Electrostatic actuation is widely used in MEMS devices to deform elastic elements [1-15]. The electromechanical response of these actuators may exhibit an inherent instability, known as the *pull-in* phenomenon [1-10]. By applying a voltage difference across the electrodes of the actuator, an electrostatic force is generated that tends to reduce the gap between the electrodes. For a sufficiently low voltage, the electrostatic force is balanced by an elastic restoring force. In this stable state the gap between the electrodes is inversely proportional to the applied voltage. Above a certain voltage, the electrostatic force is larger than the restoring elastic force for any deformation. As a result, the actuator is unstable and the gap between the two electrodes rapidly vanishes. The voltage and deformation at the onset of instability are termed *pull-in voltage* and

pull-in deformation, or in short the *pull-in parameters* of the actuator.

Several approaches for extracting the pull-in parameters have been reported in literature [1-10] and have been implemented in commercially available MEMS CAD tools [14-15]. Approximate analytical models have been suggested for electrostatic actuators [1-5]. These models yield fast results but are limited to actuators with very few degrees of freedom. To accurately calculate the pull-in parameters of general deformable elements with infinite degrees of freedom, such as beam and plate actuators, a more general approach has been suggested [4-10,14,15]. In this approach, the electromechanical response of the actuator is numerically simulated by fixing the applied voltage. The pull-in parameters are calculated by iteratively approaching the pull-in voltage with decreasing voltage increments [4-10,14,15], and henceforth this approach is referred to as the *voltage-iteration* (*VI*) scheme. This algorithm was implemented in a finite-difference scheme [4,5] and in coupled finite-elements (FEM) and boundary-elements (BEM) scheme [6-10,14,15].

In this paper, a novel algorithm for extracting the pull-in parameters of general electrostatic actuators is suggested. The algorithm is based on iterating the displacement of a pre-chosen degree-of-freedom of the actuator, rather than the applied voltage. In essence, the new Displacement Iteration Pull-In Extraction (*DIPIE*) algorithm replaces the original problem that has stable and unstable equilibrium states, with a series of equivalent problems for which the equilibrium solution is *always stable*.

The different approaches of the *VI* algorithm and the new *DIPIE* algorithm are discussed. Implementation of the new algorithm within a finite-difference code for a clamped-clamped beam actuator is described and compared with the performances of the *VI* algorithm. The comparison shows that the *DIPIE* scheme converges much faster than the *VI* scheme (>100 time faster) and it is far more consistent and well behaved.

DIPIE VS. *VI* SCHEMES

A typical static equilibrium curve of an electrostatic actuator is schematically described in Fig. 1. The

convex function describes the applied voltage as function of a representative parameter of the actuator deformation. Such a parameter may be the displacement of the center of a clamped-clamped beam. For deformations smaller than the pull-in deformation, the static equilibrium state is stable (solid line). In contrast, for deformations larger than the pull-in deformation the static equilibrium state is unstable (dashed line).
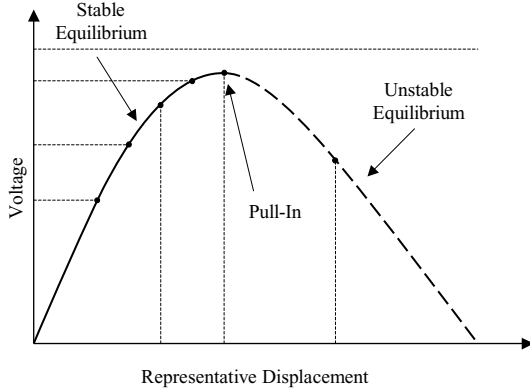

Figure 1 – Equilibrium states of the electrostatic actuator.

Two aspects of the physical response of electrostatic actuators are apparent in this figure: I. The voltage is a unique function of the deformation whereas deformation is not a unique function of voltage; II. The maximal deformation can be trivially estimated as it is bounded by the gap between the electrodes. In contrast, the maximal voltage cannot be a priory estimated.

THE *VI* ALGORITHM

In the *VI* algorithm, the pull-in voltage is iteratively approached. At each iteration, the static equilibrium deformation is calculated for the applied voltage. This calculation can be carried out by a relaxation method [6,7,9], Newton-Raphson method, or a host of other numerical schemes [6,9]. If the deformation calculation converges, it is concluded that the applied voltage is bellow the pull-in value. On the other hand, if the calculated deformation fails to converge it is concluded that the applied voltage is higher than the pull-in value. Several methods have been employed to establish whether the deformation calculation converges [4-6,9,10]. The interval between these two limits is continuously decreased until the voltage interval is smaller than a predetermined accuracy. The iterations are represented by the set of horizontal lines in Fig. 1. It can easily be seen that not all the horizontal lines cross the equilibrium curve, and therefore not all lines are associated with equilibrium states. The main advantage of the *VI* algorithm is its simplicity and ease of integration into commercial

CAD tools. For any applied voltage, the electro-elastic problem is solved by iteratively solving uncoupled electrostatic and elastic problems. It is therefore easy to implement this algorithm by sequentially employing existing numerical codes for each of these problems [6,10,14,15].
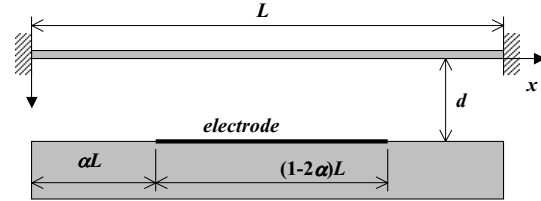

Figure 2 – The clamped-clamped beam configuration.

THE DIPIE ALGORITHM

The *DIPIE* algorithm is based on an inverse approach in which all calculations converge. At each iteration, the displacement of a pre-chosen degree-of-freedom (DOF) of the actuator is postulated. A set of reduced (voltage-free) electro-elastic coupled equations is then solved to yield the deformation of the actuator while nullifying the reaction force applied to the pre-chosen DOF. Next, the applied voltage that is required to induce the given deformation is calculated. A simple local-maximum search is then employed to iteratively approach the pull-in state where the voltage is maximal. The iterations are represented by the set of vertical lines in Fig. 1. Each of these vertical lines crosses the equilibrium curve and is therefore associated with an equilibrium state of the actuator. Like the *VI* algorithm, the *DIPIE* algorithm can be easily integrated into commercial CAD tools, using separate electrostatic and mechanical field solvers with a relaxation based algorithm.

THE CLAMPED-CLAMPED BEAM

To demonstrate the capabilities of the new scheme, the typical problem of the clamped-clamped beam is chosen [4-6]. The geometry of the problem is shown in Fig. 2. The bottom electrode is assumed at a portion of the upper beam. The equilibrium equations of the elastic beam are

$$\begin{cases} \dfrac{d^4 \widetilde{y}}{d\widetilde{x}^4} = \dfrac{1}{(1-\widetilde{y})^2}\widetilde{V}^2 & \alpha < \widetilde{x} < (1-\alpha) \\ \dfrac{d^4 \widetilde{y}}{d\widetilde{x}^4} = 0 & 0 < \widetilde{x} < \alpha \text{ and } (1-\alpha) < \widetilde{x} < 1 \end{cases}$$

where $\widetilde{y} = y/d$, $\widetilde{x} = x/L$ and $\widetilde{V}^2 = \dfrac{\varepsilon_0 W L^4}{2d^3 E^* I}V^2$.

Moreover, the effective elastic modulus, E^*, is equal to the Young modulus E if the beam thickness is of the order of the beam width, and is equal to the plate

modulus $E/(1-v^2)$ if the beam width is much larger than its thickness [5]. Also, I is the second moment of the beam cross-section, ε_0 is the permitivity of free-space, d the initial gap between the electrodes, W the width of the beam, L the length of the beam and $\{x, y\}$ are the Cartesian coordinates.

The Eqs. are iteratively solved using finite-differences in the *VI* scheme [4,5] and the *DIPIE* scheme. In each of the Pull-In search points in the *DIPIE* scheme the elastic deflection is iteratively solved using a relaxation method. In the *VI* scheme the elastic deflection is solved using the standard relaxation [4-7].
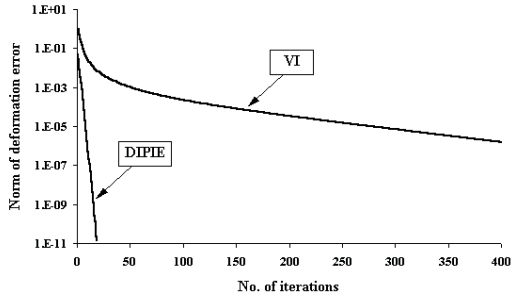

Figure 3 – Comparison between the convergence of the deformation in the *VI* and *DIPIE* algorithms.

The convergence of the elastic deflection in both schemes is described in Fig. 3. A voltage point was chosen for running the *VI* relaxation scheme. The norm of the relative error of the deflection is calculated at each of the iterations and is shown in the figure. The resulting displacement at the center of the beam is used for running the *DIPIE* relaxation scheme. The norm of the relative error of the deflection is presented in logarithmic scale against the number of iterations. It is seen that the convergence of the *DIPIE* scheme is much faster than the convergence of the *VI* scheme. Moreover, the convergence rate in the *DIPIE* scheme is constant, whereas the convergence rate in the *VI* scheme varies and declines with each iteration.

The convergence rates and the total number of iterations required to converge for both schemes at different deflections of the beam center point are described in Fig. 4. The relative error, *eu*, described in Fig. 3 is approximated by $log(eu)=a-ib$, where a is a constant, b is the convergence rate and i is the iteration number. This approximation is motivated by the linear convergences (semi logarithmic scale in Fig. 3) of the *DIPIE* scheme. Also, the convergence of the *VI* scheme tends to linearity after many iterations. It can clearly be seen that the convergence of the *VI* scheme rapidly deteriorates as the Pull-In point is approached, and that at the Pull-In point the convergence vanishes. In contrast, the

convergence rate of the *DIPIE* scheme is high and remains high even beyond the Pull-In point.
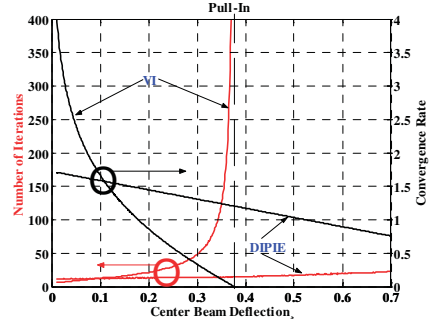

Figure 4 – DIPIE against VI schemes convergence rate and number of iterations required to converge to equilibrium states.

In order to examine the consistency of both schemes, a specific problem is solved using increasingly refined meshes for several convergence accuracies. To assure the convergence of the inner equilibrium loop, its accuracy (defined on the norm of the deflection errors) is set to two orders of magnitude higher than the required accuracy of the outer Pull-In search loop. Fig. 5 presents the Pull-In parameters calculated by both schemes against the inverse of the number of nodes. For any given accuracy, the *DIPIE* scheme shows a similar consistent convergence as the mesh is refined, making it easy to predict a value at the limit of continuum. This predicted limit converges with increasing accuracy. In contrast, the *VI* scheme shows an inconsistent behavior, which is less pronounced for the Pull-In voltage at high accuracies. It is therefore concluded that it is impractical to extract a reliable estimation of the pull-in deflection. The voltage near the pull-in state is insensitive to the deflection errors and therefore for a sufficiently high accuracy, the *VI* scheme yields reasonable estimations for the pull-in voltage.

Fig. 6 illustrates the difference in numerical effort required by each scheme to converge to the Pull-In state within a given accuracy. The numerical effort is measured by the CPU runtime required to approach the Pull-In state. The great advantage of the *DIPIE* scheme, in terms of runtime (20-120 faster), is trivially seen in this figure. In practice, due to the consistency of the *DIPIE* scheme, a lower accuracy and a coarser mesh are sufficient to extract a reliable estimation of the Pull-In parameters. On the other hand, in the *VI* scheme a higher accuracy and a finer mesh are required to reach a reliable estimation. Therefore, the advantage of the *DIPIE* scheme is even higher than appears from Fig. 6 (>100 times faster).
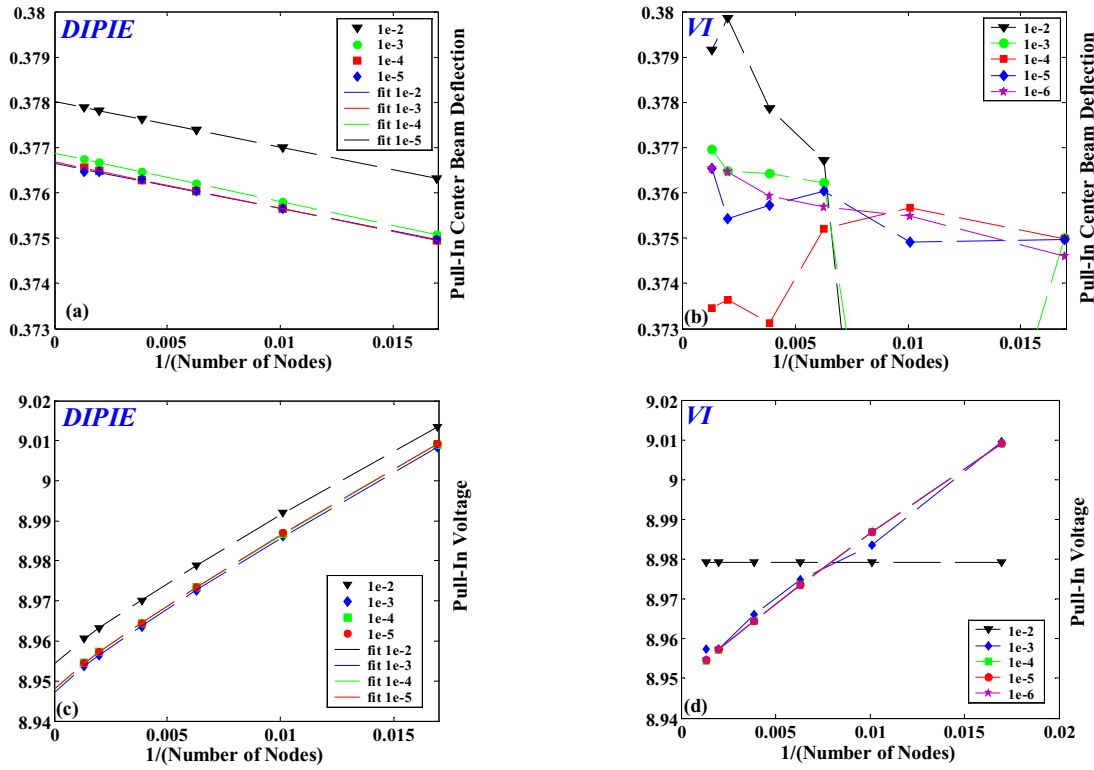
Figure 5 – Convergence with mesh and accuracy refinement for the *DIPIE* scheme (a) and (c), and *VI* scheme, (b) and (d).
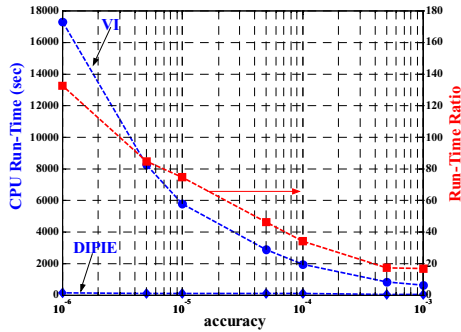

Figure 6 – DIPIE against VI schemes run-time vs. accuracy.

SUMMARY

The qualities of the *DIPIE* scheme result from the fact that the original inherently unstable physical problem of the voltage controlled electrostatic actuator is replaced by an equivalent problem that is inherently stable.

To conclude, the main advantages of the *DIPIE* scheme over the *VI* scheme are:

1. Runtime – over a 100 times faster.
2. Accuracy – consistent convergence in both accuracy and mesh refinements whereas the *VI* scheme is inconsistent and the pull-in deformation is impractical to extract.

3. Fully automated – the displacement is naturally bounded by the geometry of the actuator whereas the voltage upper bound is unknown and requires a user input.

Furthermore, the DIPIE scheme can be easily implemented in existing MEMS CAD tools using separate electrostatic and mechanical fields solvers.

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